Controlling Fano and Dicke effects via a magnetic flux in a double quantum-dot molecule

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The electronic transport through a parallel double quantum-dot molecule attached asymmetrically to leads is studied under a magnetic field. We model the system by means of a non interacting two-impurity Anderson Hamiltonian. We find that the conductance shows Fano and Dicke effects that can be controlled by the magnetic flux.

I. INTRODUCTION

Resonant tunneling through two parallel quantum dots has attracted much interest recently. For instance, Holleitner *et al.*[1] studied how the molecular states of semiconductor quantum dots connected in parallel to the leads can be coherently probed and manipulated in transport experiments, while Kubala *et al.*[2] reported a level attraction in an Aharonov-Bohm interferometer with two quantum dots in its arms. Moreover, Kang *et al.*[3] and Boese *et al.*[4] studied the double quantum dot molecule in the parallel geometry in the presence of a magnetic flux.

In Ref. 5 we reported on the transition from a series to a parallel arrangement of a quantum dot molecule attached to leads. The existence of two different pathways for the electron transport produces conductance spectra composed by a Breit-Wigner resonance and a Fano-like resonance at the bonding and antibonding frequencies, respectively. A progressive line narrowing (widening) of the Fano (Breit-Wigner) peak is observed as the system transits from the configuration in series to a symmetrical parallel one. When the symmetry is total, the Fano line shape is suppressed, indicating the cancelation of tunneling through the antibonding state, while the other peak doubles its width. The general features of the conductance spectrum taking place in the series to parallel transition of Ref. 5 are given in a parallel molecule embedded in an Aharonov-Bohm flux, as discussed by Kang *et al.*[3] and Bai *et al.*.[6] The two conductance peaks (Breit-Wigner and Fano-like curves) depend sensitively on the external magnetic field, and exhibit Aharonov-Bohm-type oscillations.

In this we consider electron transport through a parallel quantum-dot molecule embedded in an Aharonov-Bohm interferometer connected asymmetrically to leads. We show that with a period of a quantum of flux ($\Phi_0 = h/e$)the magnetic filed allows interchanging the roles of the bonding and antibonding states in the transmission spectrum. For intermediate values of the flux (namely, semi-integer multiples of a quantum of flux) the parallel molecule behaves as if it were connected in series. We also find that whenever the flux is close to integer multiples of Φ_0, the density of states shows an ultranarrow and a broad peak at the energies of the molecular states, associated to Fano and Breit-Wigner lineshapes in the conductance. When the flux has exactly the above values, the conductance experiences the suppression of the Fano line shape, indicating a localization of the corresponding molecular state, similarly to what takes place for the symmetrical case in the absence of magnetic field.[5] We find that these results hold even under a strong left-right asymmetry. This phenomenon resembles the Dicke effect in optics, which takes place in the spontaneous emission of a pair of atoms radiating a photon with a wave length much larger than the separation between them.[7] The luminescence spectrum is characterized by a narrow and a broad peak, associated with long and short-lived states, respectively. The former state, coupled weakly to the electromagnetic field, is called *subradiant*, and the latter, strongly coupled, *superradiant* state.

The appearance of the Dicke effect in resonant tunneling was first obtained by Shahbazyan and Raikh in a work on a tunneling junction with two impurities.[8] Later, Shahbazyan and Ulloa[9] studied this effect in a system of localized states in a strong magnetic field. More recently, Vorrath and Brandes[10] studied the stationary current through a double quantum dot interacting via a common phonon environment.

II. MODEL

We consider two single-level quantum dots forming a molecule, coupled to left and right leads in the way shown in Fig. 1. The interdot and intradot electron-electron interactions are neglected.
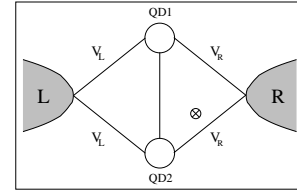

FIG. 1: Double quantum-dot molecule connected in parallel to leads

The system can be modeled by a non-interacting two-

impurity Anderson Hamiltonian

$$H = H_M + H_L + H_I. \tag{1}$$

Here H_M describes the dynamics of the isolate molecule, and is given by

$$H_M = \sum_{i=1}^{2} \varepsilon_i d_i^\dagger d_i - t_c(d_2^\dagger d_1 + d_1^\dagger d_2), \tag{2}$$

where ε_i is the energy of dot i, d_i $(d_i^\dagger)$ annihilates (creates) an electron in dot i and t_c is the interdot tunneling coupling parameter. H_L is the Hamiltonian for the non-interacting electrons in the left and right leads

$$H_L = \sum_{k_\alpha \in \{L,R\}} \varepsilon_{k_\alpha} c_{k_\alpha}^\dagger c_{k_\alpha}, \tag{3}$$

where c_{k_α} $(c_{k_\alpha}^\dagger)$ is the annihilation (creation) operator of an electron of momentum k_α and energy ε_{k_α} in the contact α. Finally, H_I accounts for the tunneling between dots and leads,

$$\begin{aligned} H_I = &\sum_{k_\alpha \in \{L,R\}} V_{1k_\alpha} d_1^\dagger c_{k_\alpha} + \text{h. c.} \\ &+ \sum_{k_\alpha \in \{L,R\}} V_{2k_\alpha} d_2^\dagger c_{k_\alpha} + \text{h. c.}, \end{aligned} \tag{4}$$

with V_{ik_α} the tunneling matrix element.

We focus in the density of states and conductance at zero temperature, which are calculated via the equation of motion approach for the Green's function[2]. The retarded Green's function is defined by

$$G_{ij}^r(t) = -i\theta(t)\langle\{d_i(t), d_j^\dagger(0)\}\rangle, \quad i,j = 1,2, \tag{5}$$

where $\theta(t)$ is the step function. The linear conductance G is related to the transmission $T(\varepsilon)$ of an electron of energy ε by the Landauer formula at zero temperature[2],

$$G = \frac{2e^2}{h}T(\varepsilon_F). \tag{6}$$

In the absence of interaction, the total transmission $T(\varepsilon)$ can be expressed as

$$T(\varepsilon) = tr\{\mathbf{G}^a(\varepsilon)\mathbf{\Gamma}^R\mathbf{G}^r(\varepsilon)\mathbf{\Gamma}^L\}, \tag{7}$$

where $\mathbf{G}^{r(a)}(\varepsilon)$ is the Fourier transform of the retarded (advanced) Green's function of the molecule, and the matrix $\mathbf{\Gamma}^{L(R)}$ describes the tunneling coupling of the two quantum dots to the left (right) lead, with

$$\Gamma_{ij}^{L(R)} = 2\pi \sum_{k_{L(R)}} V_{ik_{L(R)}} V_{jk_{L(R)}}^* \delta(\varepsilon - \varepsilon_{k_{L(R)}}), \quad i,j = 1,2. \tag{8}$$

We consider the case of quantum dots with equal energy levels: $\varepsilon_1 = \varepsilon_2 \equiv \varepsilon_0$, and denote by $|\pm\rangle$ the eigenstates of the Hamiltonian of the isolated molecule [Eq.

2], $|\pm\rangle = (|1\rangle \mp |2\rangle)/2$, with energies $\varepsilon_\pm = \varepsilon_0 + \pm t_c$. We assume that the magnitudes of the tunneling matrix elements between the dots and the left and right leads are such that $|V_{1k_L}| = |V_{2k_L}| = V_L$, and $|V_{1k_R}| = |V_{2k_R}| = V_R$, so that in the presence of a magnetic flux, they have the form $V_{1k_L} = V_L e^{-i\phi/4}$, $V_{2k_L} = V_L e^{i\phi/4}$ $V_{1k_R} = V_R e^{i\phi/4}, V_{2k_R} = V_R e^{-i\phi/4}$. Thus, in the basis $\{|-\rangle, |+\rangle\}$,

$$\mathbf{\Gamma}^{L,R} = \Gamma^{L,R} \begin{pmatrix} 1 & e^{\mp i\phi/2} \\ e^{\pm i\phi/2} & 1 \end{pmatrix}, \tag{9}$$

where we have used Eq. (8).

For simplicity, we set $\varepsilon_0 = 0$. The retarded Green's function in the same basis is

$$\mathbf{G}^r(\varepsilon) = \frac{1}{\Omega} \begin{pmatrix} \varepsilon - t_c + i\Gamma_{++}/2 & -i\Gamma_{-+}/2 \\ -i\Gamma_{+-}/2 & \varepsilon + t_c + i\Gamma_{--}/2 \end{pmatrix}, \tag{10}$$

where

$$\Omega = (\varepsilon + t_c + i\frac{\Gamma_{--}}{2})(\varepsilon - t_c + i\frac{\Gamma_{++}}{2}) + \frac{\Gamma_{-+}\Gamma_{+-}}{4}, \tag{11}$$

with $\Gamma_{--} = 2\tilde\Gamma\cos^2(\phi/4)$, $\Gamma_{++} = 2\tilde\Gamma\sin^2(\phi/4)$, $\Gamma_{-+} = i(\Gamma^L - \Gamma^R)\sin(\phi/2)$, $\Gamma_{+-} = (\Gamma_{-+})^*$, where $\tilde\Gamma = \Gamma^L + \Gamma^R$. From the diagonal elements of the Green's function we can get the spectral densities $A_\pm = -(1/\pi)\,\text{Im}\,G_{\pm\pm}^r$. Summing over the $\pm$ states we obtain the density of states of the quantum-dot molecule,

$$\rho(\varepsilon) = \sum_{\sigma=-,+} A_\sigma, \tag{12}$$

where

$$A_- = \frac{1}{\pi\Lambda}\cos^2(\phi/4)\tilde\Gamma[(t_c - \varepsilon)^2 + 4\Gamma^L\Gamma^R\sin^4(\phi/4)], \tag{13}$$

$$A_+ = \frac{1}{\pi\Lambda}\sin^2(\phi/4)\tilde\Gamma[(\varepsilon + t_c)^2 + 4\Gamma^L\Gamma^R\cos^4(\phi/4)] \tag{14}$$

with

$$\Lambda = \tilde\Gamma^2[\varepsilon - t_c\cos(\phi/2)]^2 + [(t_c + \varepsilon)(t_c - \varepsilon) + \Gamma^L\Gamma^R\sin^2(\phi/2)]^2. \tag{15}$$

The transmission, in turn, is given by

$$T(\varepsilon) = \frac{1}{\Lambda}4\Gamma^L\Gamma^R[t_c - \varepsilon\cos(\phi/2)]^2. \tag{16}$$

When $\phi = 0$ and $\Gamma^L = \Gamma^R = \Gamma_0$ (with Γ_0 the level broadening of a single quantum dot) we recover the result without magnetic field.[5] The density of states is the sum of a Lorentzian with width $\Gamma_- \to \tilde\Gamma = \Gamma_L + \Gamma_R = 2\Gamma_0$ and a delta function centered at the antibonding energy $(\Gamma_+ \to 0)$. In other words, in this limit the antibonding state is decoupled from the continuum, while the bonding state has reduced its lifetime to a half. This is due to quantum interference in the transmission through the

two different discrete states (the two quantum-dot levels) coupled to common leads. This result is totally analogous to the Dicke effect in optics, that takes place in the spontaneous emission of two closely-lying atoms radiating a photon into the same environment.[7]

The Dicke effect in the parallel molecule occurs not only for the symmetrical case, but also when $\Gamma^L \neq \Gamma^R$, and whenever the magnetic flux approaches to an integer of flux quanta. In fact, when $\phi = 2\pi n$, with n integer, the density of states takes the form

$$\rho(\varepsilon) = \frac{1}{\pi}\left[\frac{\tilde{\Gamma}}{(\varepsilon - \varepsilon_-)^2 + \tilde{\Gamma}^2}\right] + \delta(\varepsilon - \varepsilon_+), \quad \text{n even}$$

$$= \delta(\varepsilon - \varepsilon_-) + \frac{1}{\pi}\left[\frac{\tilde{\Gamma}}{(\varepsilon - \varepsilon_+)^2 + \tilde{\Gamma}^2}\right], \quad \text{n odd} \quad (17)$$

In other words, the positions of the long-lived and the short-lived states can be interchanged depending on the parity of the magnetic flux. Notice that when $t_c = 0$ the Dicke effect is still valid for $\phi = 2\pi n$, but when the system is degenerate ($\varepsilon_+ = \varepsilon_- = 0$) the narrow and the wide peaks in the density of states are superimposed, as we show below.

In order to evaluate the above expressions numerically, we set $\Gamma_L = (1 - \Delta_A)/\Gamma_0, \Gamma_R = (1 + \Delta_A)/\Gamma_0$, where Δ_A is the parameter of asymmetry. Figure 2 shows the density of states for the cases discussed above for $\Delta_A = 0.5$. Fig 2 (a) shows that density of states for $t_c = \Gamma_0$ (solid line) and $t_c = 0$ (dash line) at $\phi = 0.1\pi$. For $t_c = \Gamma_0$, we see that the narrow peak (corresponding to the "subradiant" state) developes around the antibonding state, and the broad peak (corresponding to the "superradiant" state) developes around the bonding state. In the case with $t_c = 0$, both the broad and narrow peaks are centered at $\varepsilon = 0$. For $\phi = 1.9\pi$ and $t_c = \Gamma_0$ the two peaks interchange roles as displayed in Fig 2 (b), while at $t_c = 0$ the two peak are superimposed at $\varepsilon = 0$.

Next we show that the Dicke effect is also present in the conductance. In dimensionless units conductance ($g = G/(2e^2/h)$) this quantity is given by

$$g(\varepsilon) = \frac{4\Gamma^L\Gamma^R}{\Lambda}[\varepsilon\cos(\phi/2) - t_c]^2. \quad (18)$$

In general the conductance spectrum is composed of Breit-Wigner and Fano line shapes, as shown previously by Kang et al[3] and us.[5] We note that the width of the Breit-Wigner and Fano line shapes can be controlled with the magnetic flux. In fact when the magnetic flux $\phi \to 2\pi n$, for n even (odd) the Fano line shape is associated with the antibonding (bonding) state. In the limit $\phi = 2\pi n$, the Fano line shape is suppressed and only the Breit-Wigner line shape survives. It develops around the bonding or the antibonding energies depending on whether n is even or odd, respectively,

$$g_\pm = \frac{4\Gamma_L\Gamma_R}{\left[(\varepsilon - \varepsilon_\pm)^2 + \tilde{\Gamma}^2\right]} \quad (19)$$
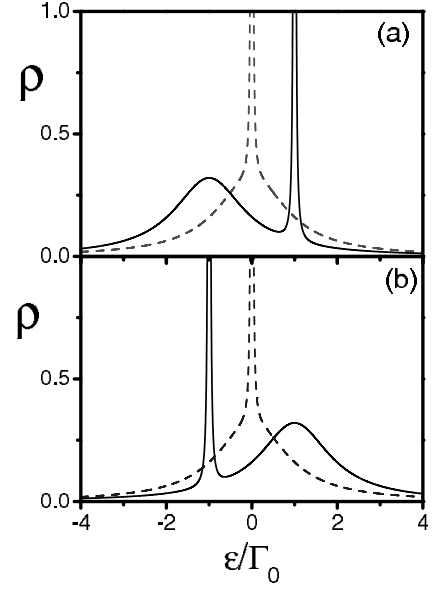

FIG. 2: Density of states ρ as a function of the Fermi energy for a) $t_c = \Gamma_0$ (solid line) and $t_c = 0$ (dash line) at $\phi = 0.1$ and b) $t_c = \Gamma_0$ (solid line) and $t_c = 0$ (dash line) at $\phi = 1.9\pi$

The above cases are displayed in Fig. 3 for $\Delta_A = 0.5$. The curves g versus Fermi energy for $\phi = 0.1\pi$ (solid line) and $\phi = 0$ (dash line) are shown in Fig 3 (a). Figure 3 (b) gives the g versus Fermi energy for the magnetic flux $\phi = 1.9\pi$ (solid line) and $\phi = 2\pi$ (dashed line). The above result can be interpreted in the following way. When the magnetic flux is an integer number flux quanta, the long-lived state is decoupled from the continuum and is supressed from transmission. For a flux close to any of these points the system would be in a regime of Dicke effect.

We consider now the special case when the quantum dots are disconnected from each other ($t_c = 0$). In this case the Fano antiresonance is localized at $\varepsilon = 0$ independent of the magnetic flux, except for $\phi = 2\pi n$. Therefore, for $\varepsilon = 0$ the conductance takes the values

$$g = 0, \qquad (\phi \neq 2\pi n, \ n \text{ integer}) \quad (20)$$

$$g = \frac{4\Gamma_L\Gamma_R}{(\Gamma_L + \Gamma_R)^2} = 1 - \Delta_A^2, \quad (\phi = 2\pi n, \ n \text{ integer}).$$

The conductance is different from zero periodically in the magnetic field, with a period of one quantum of flux. Note also that when $\phi = 2\pi(n + 1/2)$ with n integer, the Lorentzians have the same widths ($\Gamma_+ = \Gamma_+ = \sqrt{2}\Gamma$). For $\phi \to 2\pi(n + 1/2)$ (n integer), $cos(\phi) \to 0$ and hence the behavior of the conductance is reduced to a the convolution of two Breit-Wigner line shapes with the same width centered in the bonding and antibonding energies, respectively,
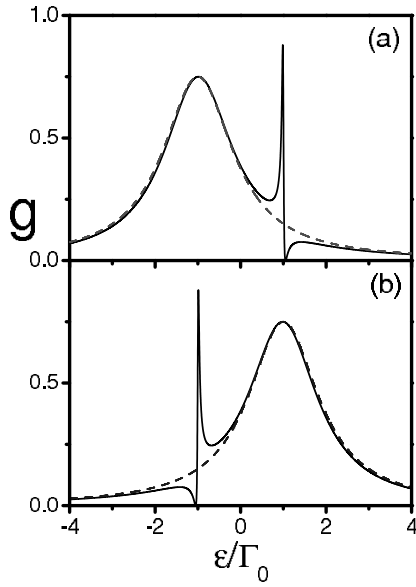
FIG. 3: Dimensionless conductance g as a function of the Fermi energy for $t_c = \Gamma_0$. (a) $\phi = 0.1\pi$ (solid line) and $\phi = 0$ (dash line), and (b) $\phi = 1.9\pi$ (solid line) and $\phi = 2\pi$ (dash line).

$$g = \frac{\Gamma_L \Gamma_R t_c^2}{[(\varepsilon - i\Gamma_L)(\varepsilon - i\Gamma_R) - t_c^2][(\varepsilon + i\Gamma_L)(\varepsilon + i\Gamma_R) - t_c^2]}. \tag{21}$$

No Fano line shape develops. The double-quantum dot in the parallel configuration behaves as a serial one for the transmission. When the electron crosses the upper (lower) arm, it accumulates a phase difference $\pi/2$ (-$\pi/2$). The contribution to the wave function of both paths interfere destructively and cancel mutually at the leads. Therefore, the paths that contribute to the conductance are only those that cross the molecule through both quantum dots sequentially, as in a serial configuration. Note that when $t_c = 0$ the conductance vanishes independently of the energy and perfect reflection is reached. A similar result was obtained previously by Kubala and König for a parallel double quantum dot system connected symmetrically to the leads.[2]

III.   CONCLUSIONS

In this work, we studied the conductance and the density of states at zero temperature of a quantum dot molecule connected asymmetrically to leads in a parallel configuration under a magnetic flux. We show that the magnetic flux can control the different regimes of conduction through the system. In particular, when the magnetic flux is close an integer number of flux quanta, the system is in the Dicke regimen. The conductance
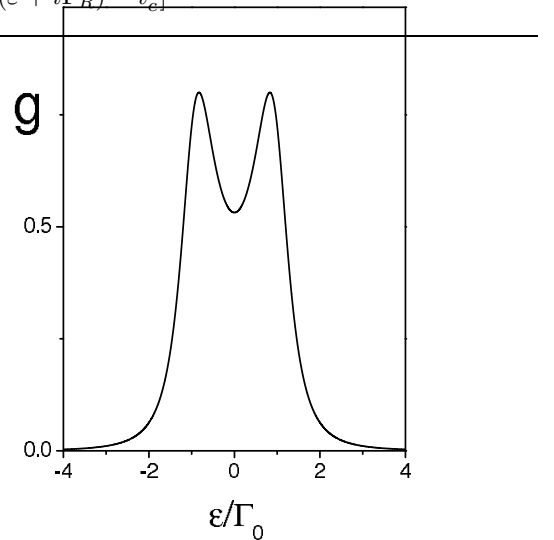

FIG. 4: Dimensionless conductance g as a function of the Fermi energy for $t_c = \Gamma_0$ and $\phi = \pi$.

spectrum is composed of Breit-Wigner and Fano line shapes at the bonding and antibonding energies, or vice versa, depending on whether this number is even or odd, with their line broadenings controlled by the magnetic flux. The narrowing (broadening) of a line in the conductance can be interpreted as an increase (reduction) of the lifetime of the corresponding molecular state. From

the densities of states it can be deduced that the antibonding (bonding) state becomes progressively localized as the magnetic flux tends to an integer number of flux quanta. When the magnetic flux is exactly a integer number of quantum flux the tunneling through the antibonding (bonding) state is totally suppressed and the bonding (antibonding) is the only participating in the transmission. Moreover, when the magnetic flux is a half integer of flux quanta, the double-quantum dot in the parallel configuration behaves as a serial one for the conductance. The control of the decoherence processes with the magnetic field exhibited by the present system may have applications in quantum computing.

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